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1979 M J ABLOWITZ AFOSR-78-3674

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The main thrust of the work has been the study of certain fundamental features of the nonlinear wave propagation which arises in the applications of many physical problems. Indeed, there are numerous disciplines in mathematics, mathematical physics and physics where these ideas have had an important impact. The work that has been accomplished relates to such diverse topics as nonlinear optics, the stability of physically important waves, special solutions to nonlinear equations, wave propagation in stratified fluids, ordinary differential equations without movable critical points etc. In part, this work has helped

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20. Abstract cont.

Clarkson College to become a center of activity in this field.

AFOSR-TR- 80-0047

ANNUAL TECHNICAL REPORT

Nonlinear Wave Propagation

AFOSR Grant 78-3674

by

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Clarkson College of Technology
Potsdam, NY 13676

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1. Research Objectives and Accomplishments

During the past year, a group of us at Clarkson College have been studying the nonlinear wave propagation associated with physically significant systems. This work has important applications in the areas of nonlinear optics i.e. self-induced transparency, certain fluid motions i.e. long waves in stratified fluids, as well as important consequences in mathematics, and mathematical physics. Below is a list of areas where we have made contributions.

(1) Nonlinear Optics

Here the application we have been interested in is the propagation of ultra short optical pulses in resonant media. This phenomena is commonly referred to as self-induced transparency (S.I.T.). S.I.T. has been intensively studied experimentally, numerically, and analytically by numerous researchers motivated by significant potential applications. Until now virtually all the mathematical analysis has been concerned with the one dimensional equations. For example, in one dimension it has been shown that arbitrary initial values break up into a sequence of coherent pulses (solitons), which do not decay as they propagate, plus radiation which rapidly attenuates. There are various types of solitons: '2π pulses' ('hyperbolic secant pulses'), '0π pulses' ('breathers') etc. We have shown, analytically that these pulses are, in fact, unstable to transverse variations. (i.e. these one dimensional waves are unstable due to multidimensional perturbations). These results are consistent with recent numerical and experimenta studies on the transverse effects of S.I.T. However, this is the first such analytical result in this direction.

(2) Painlevé Equations and Nonlinear Evolution Equations

We have discovered a deep and important connection between nonlinear wave equations solvable by inverse scattering and certain ordinary differential equations (O.D.E.'s). These O.D.E.'s have a definable mathematical property, namely they do not have any movable critical points (i.e. the singularities which depend on the initial values cannot be branch points or essential singularities). We have developed an algorithmic test to see if an O.D.E. has this property. Since Painlevé did much original work on this subject, we refer to such equations with this property as being of P type. Moreover we have found in all cases we have tested that similarity reductions of the governing nonlinear evolution equation are of P type if the evolution equation is solvable by inverse scattering.

(3) Perturbations of Solitons

We have developed a direct perturbation method which can be applied to a wide variety of problems that contain solitons and solitary waves. We have applied this method to many equations e.g. the Korteweg deVries (KdV), modified KdV, nonlinear Schrodinger, sine-Gordon, higher nonlinear KdV etc. equations. We have been able to reproduce well known results, as well as new results. Indeed, our method predicts that certain nonlinear evolution equations will have a focusing singularity. We are currently studying this question.

(4) Long Waves in Stratified Fluids

The equations which govern long internal waves in stratified fluids appear to be rather special. Recently, we have discovered a Bäcklund transformation, an infinite number of conserved quantities, and a scattering problem for these equations. In various limits we find the well known results of KdV and Benjamin-Ono. This equation is of interest to engineers and physicists.

(5) Long-time Asymptotic Solutions

Corresponding to many of these nonlinear wave equations, we can now calculate the long time asymptotic solutions both with, and without solitons. The calculations are complicated by the fact that in this asymptotic limit the solitons and continuous spectrum have different asymptotic orders. These asymptotic calculations have allowed us to develop global connection formulae for the Painlevé equations.

(6) Finite Perturbations and Solutions of Nonlinear Wave Equations

It is now well known that there are many special solutions to these wave equations e.g. solitons, rational solutions, periodic solutions, etc. We have recently developed formulae which describe, exactly, perturbations (decaying at infinity) to these solutions. Moreover, it turns out that we can describe in closed form certain perturbations which have specific properties. We refer to these solutions as Quasi-Solitons.

We are continuing our efforts on these and other problems in this area. There is considerable interest in our work and this research field in general.

2. <u>The list of publications and preprints carried out and conceived during the period of this grant are as follows</u>.

    1. Transverse Instability of One dimensional Transparent Optical Pulses in Resonant Media, Phys. Lett. 70A, #2, 83, 1979.

    2. Two-Dimensional Lumps in Nonlinear Dispersive Systems, J. Satsuma and M.J. Ablowitz, J. Math. Phys. <u>20</u>, p. 1496, 1979.

    3. A Connection Between Nonlinear Evolution Equations and Ordinary Differential Equations of Painleve Type I, M.J. Ablowitz, A. Ramani, H. Segur, accepted J. Math. Phys preprint.

    4. A Connection Between Nonlinear Evolution Equations and Ordinary Differential Equations of Painleve Type II, M.J. Ablowitz, A. Ramani, H. Segur, submitted J. Math. Phy preprint.

    5. On An Internal Wave Equation Describing A Stratified Fluid With Finite Depth, J. Satsuma, M.J. Ablowitz, and Y. Kodama, Phys. Lett. 73A, No. 4, 1979.

    6. Perturbations of Solitons and Solitary Waves, Y.Kodama and M.J. Ablowitz, preprint.

    7. Finite Perturbations and Solutions of the Korteweg deVries Equation, H. Airault and M.J. Ablowitz, preprint.

3. Professional Personnel

Professor Mark J. Ablowitz - Professor of Mathematics, Department of Mathematics, Clarkson College, Potsdam, NY 136

Dr. Yuji Kodama - Research Associate, Department of Matematics, Clarkson College, Potsdam, NY 13676.

Dr. Junkichi Satsuma - Visiting Doctoral Associate, Department of Mathematics, Clarkson College, Potsdam, NY 136

Mr. Thiab R. Taha - Graduate Student, Department of Mathematics, Clarkson College, Potsdam, NY 13676

Outside Clarkson we have collaborated with the following individuals.

Professor Martin D. Kruskal - Professor of Applied Mathematics Princeton University, Princeton, NJ 08540

Dr. Harvey Segur - Consultant, Aeronautical Research Associate Princeton University, Princeton, NJ 08540

Dr. Alfred Ramani - Research Associate, Université Paris-Sud, Orsay, France.

4. Invited Lectures, Conferences

American Mathematical Society, Syracuse University, Syracuse, NY November, 1978.

Naval Research Lab., Washington, D.C., December, 1978 (Numberical Hydrodynamics Group, Dr. David Book)

Physics Department, French Atomic Energy Commission, Saclay, France, January, 1979.

Panelist on National Science Foundation Postdoctoral Fellowship Committee, New York City, February, 1979.

Mathematics Department, SUNY Buffalo, Buffalo, NY, April, 1979

Conference on Inverse Scattering, Catholic University, Washington, DC, May, 1979.

Conference on Nonlinear Partial Differential Equation, University of Rhode Island,Kingston, Rhode Island, June, 1979.

4. <u>Invited Lectures, Conferences (continued)</u>

Conference on Solitons and Related Phenomena, Jadwisin, Poland, August, 1979.

Joint U.S.-U.S.S.R. Conference on Solitons, sponsored by the U.S. National Academy of Science and its counterpart in the U.S.S.R., Kiev, September, 1979.

Conference on Ill posed problems, University of Delaware, Newark, Delaware, October, 1979.